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DIGITALNI AKADEMSKI ARHIVI I REPOZITORIJI

# Modelling the Compost Parameters During Mechanized Aerobic Fermentation

Vjekoslav TADIĆ\*, Kurt TOMANTSCHGER, Dragan V. PETROVIĆ, Rade L. RADOJEVIĆ, Dušan Ž. RADIVOJEVIĆ

**Abstract:** The article is focused on the behaviour of the basic properties of dairy cow's solid manure over the maturation period. The manure was exposed to mechanized aerobic processing by mechanically turning-over. Performed analysis comprehended measurements of mass participation percentages of organic components, mineral matters, H<sub>2</sub>O, N, P<sub>2</sub>O<sub>5</sub> and K<sub>2</sub>O, as well as measurements of PH-values during the seven weeks of supervised test period of effective aerobic preparation of the composting material. Through maturation, large growth of averaged mass percentages of nitrogen, phosphorous-pentoxide and potassium-oxide, more than triple in comparison to the initial levels in a fresh solid, has been verified. Following a possible common approach in the scientific and technical practice, an analytical semi-empirical approach was used in describing the changes over the time of manure properties during composting, supported by prototype of the machine Kompomat 1. Combining the analytical theory of diffusion partial differential equations and application of numerical methods to non-linear fitting of the experimental output data, it was verified that all acquired manure parameters follow an identical general exponential law ( $R^2_{\text{all properties}} = 0,906 - 0,990$ ;  $RMSE_{\text{all properties}} = 0,010 - 2,544$ ). Clear relationships between the values of basic parameters of solid manure under mechanized aerobic fermentation ( $y$ ) and composting duration lap time ( $x$ ) have been established.

**Keywords:** compost; cows machine; diffusion equation; fertilizer; manure; maturation

## 1 INTRODUCTION

Today in highly intensive agriculture production, increasing attention is focused on renewal energy resources and environmental standards. Thus, in intensive livestock farming, large amounts of animal's manure are remaining for composting, which can be further used in either conventional or organic farming. Under deterioration of environmental conditions and widely recognized common legislation, contemporary animal production demands extremely careful management. Under un-adequate control, it obviously suffers from the pollution of the ground fresh water and drinking water with nitrates, pathogens expansion, excess of animal nutrients in the drinking water, increased biological oxygen consumption, and sediments formation in surface fresh water flows [1]. To reduce impacts of these appearances, precautions should be undertaken carefully. One of possible approaches of this kind is related to composting, as a process that should achieve the following goals [2]: (a) high efficient decomposing of involved organic matters, where possible, into a stable products giving a final material suitable for soil amelioration or to achieve other benefits; (b) transformation of waste material into one or more useful semi-finished or final products (composting as a process may be economically and ecologically favourable with respect to alternative disposal approach) (c) sanitizing, bio-remediation and/or bio-degradation of hazardous pathogenically polluted organic wastes to safe materials that may find further beneficial applications. This statement is also recognized by Lončarić et al. [3] who introduce the term of ecological index.

According to Pergola et al. [4, 5] compost generally is some kind of sanitized and stabilized biological material, applicable in the restoring processes of less or more degraded agricultural soils. In addition, it can strongly contribute to various technologies for recovering soil fertility, decreasing the carbon sequestration in arable land, and consequently the cut down of various chemical contaminants such as the fertilizers, pesticides, fuel, etc., and overall reduction of the production costs and environmental pollution. Pergola et al. [5] also claimed that

excessive metabolic waste effluents of domestic cattle could impose complex problems for livestock farm owners, related to their management, safety of disposing and strategic breaking down the environmental impacts by composting

Present study is primarily focused to formulation of a semi-empirical analytical model specified to facilitate predicting the changes over the time of basic parameters that characterise mechanical aerobic composting process of dairy cow's solid manure, accomplished by prototype of the self-propelled machine Kompomat 1. The aerobic process itself, as well as the machine, was explained in details by Radojevic & Topisirović [6]. In general, the composting procedure of this kind represents a simple possible approach in natural fertilizer production, among a wide variety of different existing techniques, which enables cut down of the hazards arisen under inadequate management of cattle manure. Inappropriate manure treatment most obviously results in output of poor quality, having physical and nutritive characteristics under the optimal, unpredictable non-homogeneous properties and quality, high level of mechanical contaminants, etc.

However, the aerobic composting has many important advantages. It facilitates the overall manure manipulation process, reduces maturation period from 260 - 345 days to 45 - 55 days (depending primarily on the ambient temperature), strongly decreases the overall energy consumption and simultaneously provides a product of high nutritional and ecological quality [7].

Arriaga et al. [8] analysed gaseous losses of ammonium and green-house-gasses, based on the data acquired (after turning sessions) from composting windrows placed at six different farm locations in northern Spain. They concluded that the intensity of these losses is related primarily to the moment of windrow stacking.

In the situations when the composting process of cattle manure includes application of wheat straw filler, the separate analysis of the properties of both components is needed, as well as when they are merged together. Researching the relationships between their properties is crucial for successful compost production. The ratio between the mature and wheat straw filler is directly

proportional to the quantity of the straw applied to form the litter. Amount of straw used in the aerobic composting process is also dependent on the carbon to nitrogen (C:N) ratio that optimally should be between the ranges 18:1 and 20:1. The approach of such kind intensifies breeding and growth of aerobic microorganisms, which develop a high temperature enabling break down of organic matter [9].

Some other authors, like Vukobratović et al. [10, 11] also stated that carbon to nitrogen (C:N) ratio was stabilized to 20:1 in the final composting product, but they suggested that the best data for manure maturity is final and initial C:N ratio as well as  $\text{NH}_4 - \text{N}/\text{NO}_3 - \text{N}$  ratio. During maturation, chemical as well as the physical (temperature) properties of manure are changing. Changes of temperature can be divided into heating phase, thermophilic phase and cooling phase. All these properties as well as changes of chemical properties are analysed in this paper, because all these phases of changes have some mathematical properties, which can be parameterised digitally. Thus, this paper is explaining changes of manure properties during maturation with mathematical model according to exponential function.

According to Chandra & Singh [12], the essence of mathematical modelling of physical processes gives the advantage of analysing interrelations of experimental variables among themselves. Thus an elaborate, time-consuming and expensive experimentation is not necessary every time. The important influence of the manure on the environment motivated many researchers worldwide to attempt to formulate various mathematical models in this area. In the fertilizers of all kinds, it is crucial to preserve nitrogen, because of its great mobility. This was the main motive for Hengnirun et al. [13] to present their model for simulation of ammonia volatilization in manure and soil. Same author [14] presented similar model of one-dimensional dynamic model, which simultaneously simulates nitrogen transformations (volatilization, net minerals immobilization, and de-nitrification), nitrogen transport (runoff and leaching), plant uptake, and adsorption in homogeneous, unsaturated soils.

Oudart et al. [15] claimed that "semi-empirical models are suitable, considering the scarce data available in farm situations". Processing and analysing the raw data continuously measured during the composting of eleven pigs or poultry manure static piles, they evidenced and observed the most important processes responsible for control of transformations and loss of nitrogen. The same authors have also formulated and implemented a novel mathematical model describing phenomena of mutual interactions between the nitrogen (N) and carbon (C) during composting the naturally ventilated homogeneous piles containing animal manure. In some cases, but especially those of special interest, the fitting constants of such models were calibrated using available (acquired) sets of experimental data and non-linear fitting methods, what resulted in development of appropriate so-called semi-empirical model. Authors concluded that multiplication and growth of various kinds of microbes could influence decreased evaporation of ammonia ( $\text{NH}_3$ ) in environmental atmosphere and noticed that greatest conservation of nitrogen was evidenced at limited reproduction of microbes caused by nitrogen availability.

Mason [16] explored the applicability of various mathematical and physical models for predicting the scalar fields of temperature in reactors engaged in the composting process and reported that existing physical models often poorly describe the full-scale composting volume parameters and its environment. Consequently, they could generate inappropriate outputs of simulation, which usually leads to wrong conclusions at full scale (real) situations. More precisely, the problem is mainly caused by large differences between the scales of laboratory and real composting reactors. Laboratory self-heating reactors are much smaller with respect to production real-scale reactors and, therefore, obviously exposed to high losses of heat, even when insulated. To overcome this problem, the same authors suggested future laboratory application (introduction) of more advanced small-scale test-reactors, possessing special equipment for control of the temperature difference (or heat flux) of laboratory reactors.

The great importance of this area of agriculture is shown in the paper of Walling et al [17] which reviewed 40 years of literature related to mathematical models of various composting processes. After thorough analysis of acquired published data, these authors concluded that models of this kind are mostly sensitive toward three factors: the ratio between birth/growth and dying rates of microbes actively involved in the composting process; the rates of consumption and product yields.

At present, various approaches exist for analysis, evaluation and predicting the animal manure properties in different transformation phases of the aerobic treatment. Simultaneously, the modern mathematical physics rely, among many others, on the theory of parabolic-type partial differential equations. There are many reasons that motivate researchers and excite their consistent interest to advance the solving methods of parabolic equations. However, their attention is initiated primarily by the profoundness and complex structure of the mathematics involved, as well as by its capabilities in resolving a wide variety of different problems in natural sciences, technology, and economics [18, 19].

In a way, present paper links the two distant scientific areas: livestock waste material composting and analytical theory of partial differential equations. It presents formulation and experimental testing of a simple mathematical model intended to describe analytically the changes over the time of a dairy cow's manure mixed with straw litter filler during composting-aerobic treatment.

## 2 MATERIALS AND METHOD

During the last century, a variety of different analytical and numerical methods have been derived with intention to facilitate resolving of existing technical problems based on the mathematical models that include ordinary or partial differential equations. Unfortunately, most of the methods of this kind could be applied successfully only at academic level. However, some techniques that contribute in resolving the practical scientific and technical problems have been developed [20]. In a way, present study is oriented toward reduction of this imbalance. Thus, an appropriate parabolic second order partial differential equation (diffusion Eq. (1)):

$$\frac{\partial^2 y}{\partial x^2} + \left[ \frac{b+2cx}{2(a+bx+cx^2)} - \frac{2c}{b+2cx} \right] \frac{\partial y}{\partial x} + \frac{(b+2cx)^2}{a+bx+cx^2} \frac{\partial y}{\partial t} = 0 \quad (1)$$

is applied as a starting point in developing general semi-empirical model function, which describes the changes over the time of dairy cow's manure parameters during the composting.

After adequate analytical transformations and introduction of appropriate additional conditions, the diffusion equation is solved providing the exact analytical solution in the form of exponential model function specified for description of changes over the time of manure parameters during composting. Constants of exponential function were determined by non-linear fitting, separately for each manure property. Therefore, our analytical model belongs to a class of so-called inductive models [21], inspired by the measured data. Verification of the model was performed comparing own experimental data (from a pile of dairy cow's manure with straw filler, exposed to aerobic processing under support of the composting machine prototype Kompomat 1) with their appropriate values (counterparts) fitted by exponential model function. Time variations of the characteristics of cow's manure mixed with wheat straw filler, during aerobic composting fermentation, have been considered.

During the eight weeks of composting, the most important properties were continuously observed, including: temperature, mass participations of  $H_2O$ , organic and mineral matters, total and soluble N,  $P_2O_5$  and  $K_2O$ , together with manure pH-values. Initial parameters of the manure at the beginning of the process are given in Tab. 1.

Samples of the compost, i.e. residue after each specified phase of fermentation, were exposed to homogenization with an electric mixer and the acidity of the residue was measured through whole maturation process with pH meter. The dry matter content was determined by drying up-to the constant weight, and the content of organic matter was estimated using measured values of the ash content in the dry matter of the acquired samples. The mass participation of the ash was determined by burning the compost samples material at  $550^\circ C$  in the furnace. The total nitrogen was measured by the Kjeldahl method, and the phosphorus by vanadate-molybdate method spectrophotometry. The contents of other tested elements were determined using an atomic absorption spectrometer, after dry burning of the samples. As it was logical and expected, water content varied in different samples of the compost, especially those acquired after different periods over the time of aerobic processing.

Components have been merged and mixed using a standard approach, in the channels for the manure cleaning, placed in the dairy barn: raw manure was spread out over the concrete plateau in the form of prismatic piles of 3 m in width, having height of 1 m. Depending on the manure quantity, which was between 770 and 830 t/day, pile lengths were between 75 m and 85 m - 80 m in average, Fig. 1c. However, length of the prisms does not influence the process itself.



Figure 1 The self-propelled machine Kompomat 1: (a) prototype view; (b) untreated (raw) pile; (c) machine in work pile treatment

Mechanical turning-over of the cow's solid manure and wheat straw filler mixture has been done periodically during two months, using a prototype machine Kompomat 1 (Fig. 1a and Fig. 1b): three times during the first seven days, and two times per week in the next 35 days. During the last two weeks (seventh and eighth), mechanical turning over was not applied. Depending on the sequential order of treatment and manure condition, operational speed of Kompomat 1 was set to the values from 0,1 km/h to 0,7 km/h. Basic components of this machine prototype are sketched in Fig. 2 (1 - rotor, 2 - transmission with bearings, 3 - power assembly of the roller, 4 - tunnel, 5 - redesigned housing of the side reductor, 6 - chassis, 7 - drive wheels, 8 - wheels for steering, 9 - redesigned steering mechanism, 10 - front protection rubber, 11 - rear protection rubber panel, 12 - machine power assembly, 13 - driver's cab.

Output shaft of machine transmits the power to the working element, through single stage chain reductor. Radius of working element is fixed: 0,328 m, while rotation rate varied in the range of 300 - 500 o/min. Measurements of torque have been performed by measuring system TRC-MMN1 and sensor 2 kNm TD2, supported with oscilloscope Tektronix 2230. Manure rupture resistance was  $2000 \text{ daN/m}^3$  and friction resistance of working element was  $1000 \text{ daN/m}^3$ . Load of working implements, originating from the manure, was determined analytically and reached  $4000 \text{ daN/m}^3$ .

Similar composting machines are used in [24]. Also, modern machine learning techniques are used for management of outdoor compost piles [25].

Values of all measured characteristics of dairy cow's manure were fitted toward composting time assuming the exponential function.

$$y = e^{a+bx+cx^2} \quad (2)$$

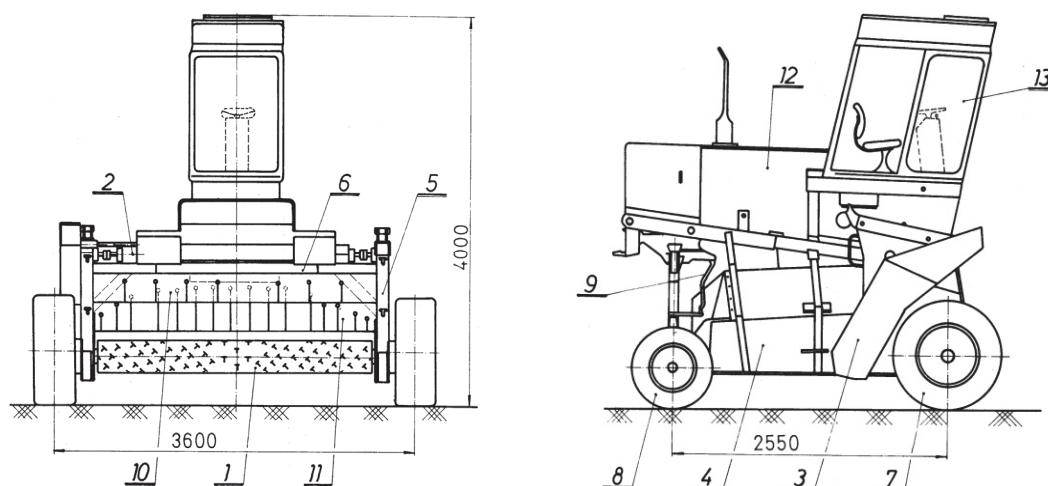


Figure 2 Basic elements of Kompomat 1 machine

The least-squares fitting method was used to adjust the values of  $a$ ,  $b$  and  $c$  [22, 23]. All computations are performed using the software package *R* (Institute for Statistics and Mathematics) and SAS Enterprise Guide 7.1. The accuracy of performed fitting procedures was evaluated according to the *R*-square values, adjusted *R* square values and root-mean-square (standard) errors *RMSE/SE*. Their values are given in Tab. 2.

### 3 RESULTS AND DISCUSSION

#### 3.1 Formulation of the Analytical Model

The problem consideration is based on the expression (2). It is founded on the diffusion Eq. (1), where the unknown function  $y(x, t)$  is the specific compost property, and  $x$  denotes the length of fertilising period (number of days passed from the beginning of the aerobic process). In this chapter is presented the solving algorithm of the partial differential Eq. (1), which gave the Eq. (2). The differential Eq. (1) must be written in the general form of a homogeneous diffusion equation:

$$M(x) \frac{\partial y}{\partial t} = \frac{\partial}{\partial x} \left[ N(x) \frac{\partial y}{\partial x} \right] \quad (3)$$

By differentiation of the right side of Eq. (3), expressing the second derivative with respect to  $x$  explicitly and then equating the factors of  $\frac{\partial y}{\partial x}$  and  $\frac{\partial y}{\partial t}$  with those of Eq. (2), one receives the equations of determination for  $M$  and  $N$ . They are:

$$\frac{N'}{N} = \frac{b+2cx}{2(a+bx+cx)^2} - \frac{2c}{b+2cx}, \quad ' = \frac{d}{dx'} \quad (4)$$

and,

$$\frac{M}{N} = - \frac{(b+2cx)^2}{(a+bx+cx^2)} \quad (5)$$

Integration of Eq. (4) yields:

$$N(x) = c_1 \frac{\sqrt{a+bx+cx^2}}{b+2cx} \quad (6)$$

$c_1$  is an arbitrary constant of integration. Inserting  $N$  into Eq. (5) gives:

$$M(x) = -c_1 \frac{b+2cx}{\sqrt{a+bx+cx^2}} \quad (7)$$

Thus, it was shown that Eq. (1) can be written as a diffusion Eq. (3) which possesses the form:

$$\frac{b+2cx}{\sqrt{a+bx+cx^2}} \frac{\partial y}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{\sqrt{a+bx+cx^2}}{b+2cx} \frac{\partial y}{\partial x} \right] \quad (8)$$

$c_1$  was cut away.

Since neither a solution nor a calculation method for the diffusion Eq. (3) could be found to this day, each equation has to be solved separately. To get an idea of what the solution  $y(x, t)$  of Eq. (1) might look like, consider the simpler one-dimensional heat-flow equation:

$$\frac{\partial y}{\partial t} = \beta \frac{\partial^2 y}{\partial x^2} \quad (9)$$

It describes the heat-flow in a row whose surface is thermally insulated. The function  $y(x, t)$  represents the temperature distribution at point  $x$  for the time  $t > 0$ .  $\beta > 0$  is a constant. A particular solution is:

$$y(x, t) = \frac{1}{2\sqrt{\beta\pi t}} \exp\left(-\frac{x^2}{4\beta t}\right) \quad (10)$$

The goal is to find a unique solution of the partial differential Eq. (1). Since Eq. (1) is more complicated than Eq. (9), it can be expected that the solution of Eq. (1) resp. Eq. (8) will be more complicated than Eq. (10). Therefore, the solution of Eq. (1) is supposed to be the following function:

$$y(x, t) = C(x)t^p \exp(F(x)t^q) \tag{11}$$

Constants  $p$  and  $q$ , different from zero, need to be determined, but for the functions  $C(x)$  and  $F(x)$  to be calculated,  $F(x)$  is of no interest as a constant. Expressing  $\frac{\partial y}{\partial t}$  from Eq. (1) resp. Eq. (8) explicitly and inserting the attempt Eq. (11) into the resulting equation yields the non-linear condition equation:

$$\begin{aligned} \frac{pC}{t} + \left[ \frac{1}{2(b+2cx)} - 2c \frac{a+bx+cx^2}{(b+2cx)^3} \right] C' + \frac{a+bx+cx^2}{(b+2cx)^2} C'' + q C F t^{q-1} + \\ + \left[ \frac{1}{2(b+2cx)} - 2c \frac{a+bx+cx^2}{(b+2cx)^3} \right] C F' t^q + \frac{a+bx+cx^2}{(b+2cx)^2} [2C' F' + C F''] t^q + \\ + \frac{a+bx+cx^2}{(b+2cx)^2} C F'^2 t^{2q} = 0 \end{aligned} \tag{12}$$

This is only one (ordinary differential) equation for four unknowns  $C(x)$ ,  $F(x)$ ,  $p$  and  $q$ . But since  $x$  and  $t$  are independent variables a comparison of coefficients with respect to  $t$  powers can be done. This creates several equations of determination.

### 3.2 The First Option for Solution

If the unknown functions  $P(t)$  and  $Q(t)$  had been used instead of the powers  $t^p$  and  $t^q$  in assumption of the solution Eq. (11), the coefficient comparison according to  $t$  powers in Eq. (12) could not have been made (carried out). But since both in partial differential Eq. (1) to be solved, as in Eq. (9) the variable  $t$  only occurs as a derivative and is not in the coefficients, it is very obvious that in the solution of Eq. (1) resp. Eq. (8) the variable  $t$  also occurs as a power as in Eq. (10).

Furthermore it is assumed that  $q$  is also an integer as in Eq. (10). Then all  $q$ -powers occurring in Eq. (12) would be different from each other and so all factors of  $t^{-1}$ ,  $1$ ,  $t^{q-1}$ ,  $t^q$  and  $t^{2q}$  would be zero. Such solutions would be e.g.  $C = 0$  or  $F = \text{const.}$  They are uninteresting and of no use. This is the case for  $|q| \geq 2$ . Therefore, only the cases  $q = 1$ ,  $q = 0$  and  $q = -1$  remain.

#### 3.2.1 Case 1: $q = 1$

In this case there are the four different  $t$  powers  $t^{-1}$ ,  $t^0 = 1$ ,  $t$  and  $t^2$  for three unknowns. So the system of equations is overdetermined. The factors of  $t^{-1}$  and  $t^2$  provide the determination equations:  $p C = 0$ ,  $C F' = 0$ .

Since  $C = 0$  yields  $y = 0$  the factors of  $C$  vanish. This implies  $p = 0$  and  $F = \text{const.}$ , which cannot be used in application.

#### 3.2.2 Case 2: $q = 0$

In this case there are only the two different  $t$  powers  $t^{-1}$  and  $t^0$  for the three unknown  $p$ ,  $C(x)$  and  $F(x)$ . E.g. this is the underdetermined case because the number of equations is fewer than the number of unknowns. Similarly as before, taking the factor of  $t = -1$  yields  $pC = 0$ , i.e.  $p = 0$  which implies  $y = y(x)$ . This solution is not applicable.

#### 3.2.3 Case 3: $q = -1$

And as before: in order that the Eq. (12) is satisfied for all values of  $t$ , it is necessary and sufficient that the factors of each power of  $t$  vanish. Thus the coefficients of  $t^{-2}$ ,  $t^{-1}$ ,  $1$  yield three equations for the three unknowns  $p$ ,  $C(x)$  and  $F(x)$ :

$$F - \frac{a+bx+cx^2}{(b+2cx)^2} F'^2 = 0 \tag{13}$$

$$pC + \left[ \frac{1}{2(b+2cx)} - 2c \frac{a+bx+cx^2}{(b+2cx)^3} \right] C F' + \frac{a+bx+cx^2}{(b+2cx)^2} [2C' F' + C F''] = 0, \tag{14}$$

$$\left[ \frac{1}{2(b+2cx)} - 2c \frac{a+bx+cx^2}{(b+2cx)^3} \right] C' + \frac{a+bx+cx^2}{(b+2cx)^2} C'' = 0$$

Eq. (13) represents a non-linear differential equation. Such equations are not easy to solve because there are very few types of such equations for which solving methods could be found till today. But by taking a closer look at Eq. (13), one sees that:

$$F' = b + 2cx \tag{15}$$

It implies:

$$F(x) = a + bx + cx^2 \tag{16}$$

which satisfies Eq. (13) indentially. A simple particular solution of Eq. (15) is  $C = \text{constant}$ . Putting  $C$  and  $F(x)$  in the determination Eq. (14) gives:

$$p = -\frac{1}{2} \tag{17}$$

Inserting  $q = -1$ , Eq. (15) and Eq. (17) access in Eq. (11) which yields the propability density function:

$$y(x, t) = \frac{C}{\sqrt{t}} \exp\left(\frac{a+bx+cx^2}{t}\right) \tag{18}$$

where  $C$  is an arbitrary constant.

### 3.3 The Second Option for Solution

The power of this solving method is that it turns the partial differential Eq. (1) into the ordinary differential Eq. (12) which is the goal of almost all solution methods for partial differential equations. The disadvantage of this solving process applied in this paper is that the ordinary differential Eq. (12) resp. Eq. (13) is not linear. Taking  $C = 1$  and the initial condition  $t = 1$  then Eq. (18) becomes the particular solution:

$$y(x, t=1) = e^{a+bx+cx^2} \tag{19}$$

But that is also the fitting function with respect to the length of fertilising period as independent variable. In this the applicability of the exponential function in approximating the manure fertilising distribution has been successfully verified. In addition, the mathematical model (partial differential equation) has been also experimentally verified.

### 3.4 Experimental Data Fitting

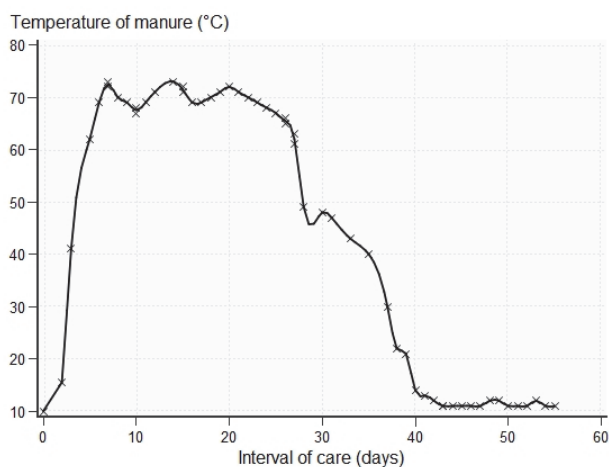
Initial manure characteristics, just at the forming of the prismatic piles, are listed in Tab. 1. These data, for different compost characteristics, represent the first "fitting points" at the charts presented in Fig. 3, Fig. 4 and Fig. 5.

**Table 1** Manure parameters at the beginning of composting process

Water / %	Mineral matter / %	Organic matter / %	pH	N / %	P <sub>2</sub> O <sub>5</sub> / %	K <sub>2</sub> O / %
79,70	0,45	19,85	7,62	0,62	0,32	0,45

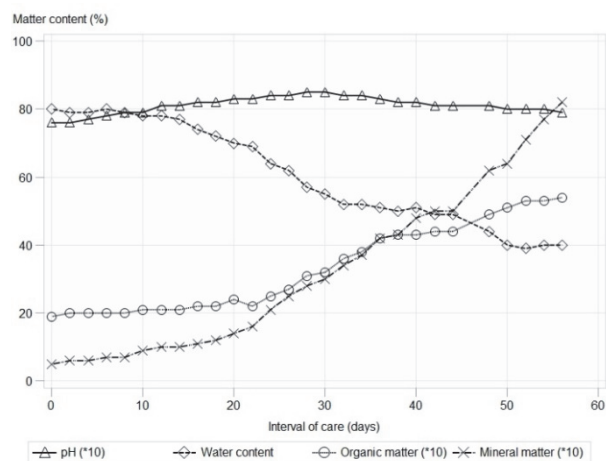
N; P<sub>2</sub>O<sub>5</sub> and K<sub>2</sub>O are expressed with respect to dry matter content (mineral matter + organic matter).

Fig. 3 presents temperature changes of manure during aerobic processing. This temperature reached the maximum value of about 70 °C after only 4 days and maintained it during the first 3 weeks. The manure temperature stabilized its value after about 52 days (temperature 12 - 15 °C), indicating the end of the active fermentation process.

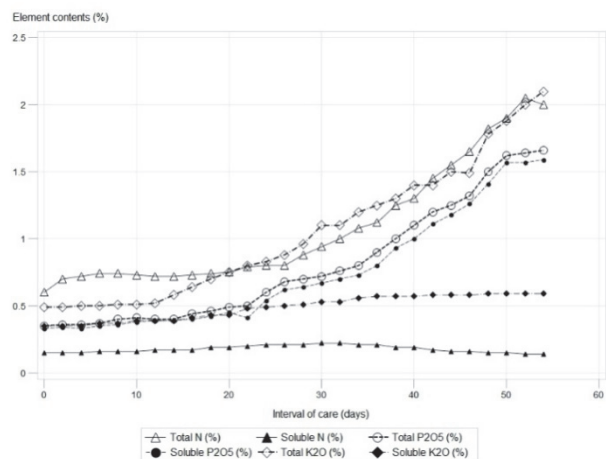


**Figure 3** Temperature of manure during the aerobic processing

Under aging caused by composting process of intersets, the convergence over the time of the manure characterisation parameters generally followed the exponential law having the exponent of second-order polynom, described by Eq (1) see Fig. 3, Fig. 4, Fig. 5. This finding is illustrated in Fig. 3, Fig. 4, Fig. 5 which verify agreement between the measured and fitted values of manure properties of interest.



**Figure 4** Changes over the time of matters contents during the aerobic processing of dairy cows' manure



**Figure 5** Changes over the time of elements contents during the manure aerobic processing of dairy cow's manure

**Table 2** Fitting constants and quality (i.e. accuracy) characterization parameters

Property / %	<i>a</i>	<i>b</i>	<i>c</i>	<i>R</i> <sup>2</sup>	<i>R</i> <sup>2</sup> <sub>adj.</sub>	<i>RMSE</i>
Water content	4,38025	-7,03 10 <sup>-5</sup>	-2,58 10 <sup>-4</sup>	0,990	0,989	1,47881
Mineral matter content	0,8527	0,10223	-6,66 10 <sup>-4</sup>	0,992	0,991	2,54407
Organic matter content	2,85637	0,02475	-6,89 10 <sup>-5</sup>	0,978	0,977	1,78894
pH	4,32371	0,00555	-7,41 10 <sup>-5</sup>	0,951	0,948	0,59877
Total N	-0,54695	0,02041	6,8804 10 <sup>-5</sup>	0,970	0,967	0,08831
Soluble N	-2,02645	0,01918	-6,4179 10 <sup>-4</sup>	0,906	0,899	0,01014
Total P <sub>2</sub> O <sub>5</sub>	-1,44663	0,04178	-6,57 10 <sup>-5</sup>	0,981	0,980	0,07223
Soluble P <sub>2</sub> O <sub>5</sub>	-1,61623	0,04072	-1,17 10 <sup>-5</sup>	0,986	0,985	0,05899
Total K <sub>2</sub> O	-0,88289	0,02276	1,44 10 <sup>-4</sup>	0,987	0,986	0,06435
Soluble K <sub>2</sub> O	-1,45846	0,03142	-2,15 10 <sup>-4</sup>	0,948	0,9446	0,03469

Values of the fitting constants *a*, *b* and *c* of the model Eq. (1) are listed in Tab. 2, together with the *R* square

values, adjusted *R* square values and root-mean-square errors (*RMSE*) so called standard errors (*SE*).

The fitting quality was excellent. The  $R$  square values, adjusted  $R$  square values reached high values of 0.9 or higher, verifying that the model Eq. (1) is adequate for describing the aerobic processing of dairy cows manure. The  $RMSE$  ( $SE$ ) values were small, additionally verifying the applied fitting Eq. (1). The values of these fitting parameters are given in Tab. 2. All mathematical calculations and statistical models were done with [26, 27].

#### 4 CONCLUSION

Modelling of a wide variety of different techniques applied in actual agricultural engineering has an important role. Consequently, in this study is presented a semi-empirical (inducing) mathematical model, which includes an appropriate diffusion partial differential equation, solved in an original way. The coefficients of exponential Eq. (1), which represents its analytical solution, are determined by fitting the experimental data. These data were provided by observing the convergence over the time of dairy cow's manure properties, exposed to the aerobic fermentation under the support of mechanized turning-over of the solid manure with a prototype machine Kompomat 1.

During composting supported by mechanized turning of the manure, the mass participation of nitrogen, phosphorous-pentoxide and potassium oxide, raised by over 300% in average in relation to the manure condition at the beginning of the fermentation process, has been registered. However, it was confirmed experimentally that variations of all variables of interest generally follow the same form of exponential law  $y = e^{a+bx+cx^2}$ , where independent variable  $x$  is the time measured from the beginning of the fermentation period. Primary advantage of empirical data fitting lies in the fact that it enables simple description of a process of interest with an equation comprehending just a few fitting constants, instead of a set of experimental data represented by appropriate numerical values. These fitting constants (coefficients) in the model function,  $y = e^{a+bx+cx^2}$  proposed in present study, were determined by non-linear fitting.

Therefore, verified exponential function can be successfully applied to predict the values of manure properties in advance, before initiation of the aerobic composting. Furthermore, these kinds of fitting functions facilitate further formulations and improvements of mathematical models specified for description of more sophisticated processes that comprehend composting of the solid manure and its application.

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